

# Integrating Scientific Theory with Machine Learning

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- Introduction
- Merging mechanistic and machine learning models
  - How is the knowledge represented?
  - Where is the knowledge integrated in the machine learning pipeline?
  - How is the knowledge integrated [1]?
- Integrating state-space and deep learning models
- Some works on hybrid models in bioreactors

- Please note that my group is not in favor of black-box learning
- We are interested in:
  - generative and probabilistic models,
  - integrating physical models into ML,
  - working with time series (not i.i.d) data,
  - quantifying uncertainties in predictions and classifications,
  - providing some explainability.

## Machine learning

- Not enough data to train sufficiently generalized models.
- Purely data-driven model might not meet constraints such as dictated by natural laws, or given through regulatory guidelines.
- Machine learning models becoming increasingly complex -> need for models to be interpretable and explainable
- Require homogeneous labeled training data

## Mechanistic models

- “A picture is worth a thousand words” -> “A model is worth a thousand datasets.”
- Normally, cannot capture complex dynamics in the system
- Often, they are simplified to allow for handling complexity

## Hybrid models

- Best of both worlds
- Can be complex and difficult to train

- *General knowledge*: knowledge independent of the task and data domain.
- *Domain knowledge*: knowledge in any field such as physics, chemistry, engineering, and linguistics with domain-specific applications.






Algebraic Equations	Logic Rules	Simulation Results	Differential Equations	Knowledge Graphs	Probabilistic Relations	Invariances	Human Feedback
$E = m \cdot c^2$ $v \leq c$	$A \wedge B \Rightarrow C$		$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ $F(x) = m \frac{d^2 x}{dt^2}$				

Figure: Domain knowledge representation [1]<sup>1</sup>

<sup>1</sup>Note that all figures are copied and the sources are referenced!

*Informed machine learning* describes learning from a hybrid information source that consists of data and prior knowledge. The prior knowledge is pre-existent and separated from the data and is explicitly integrated into the machine learning pipeline [1].

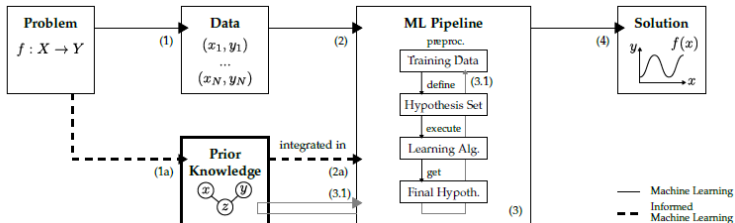
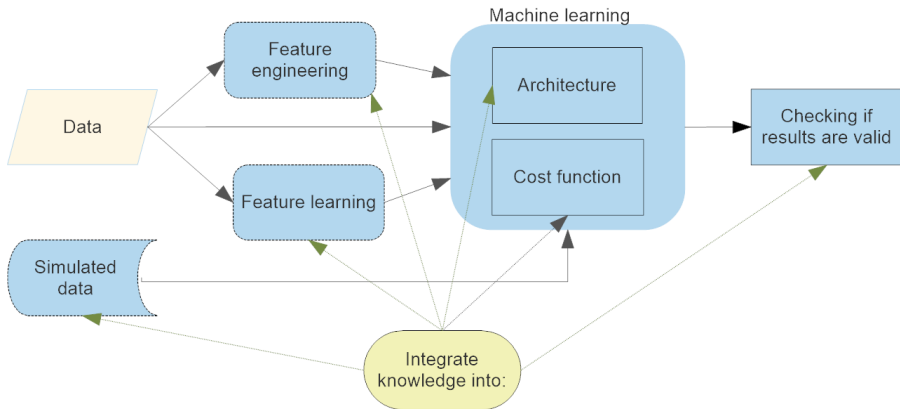
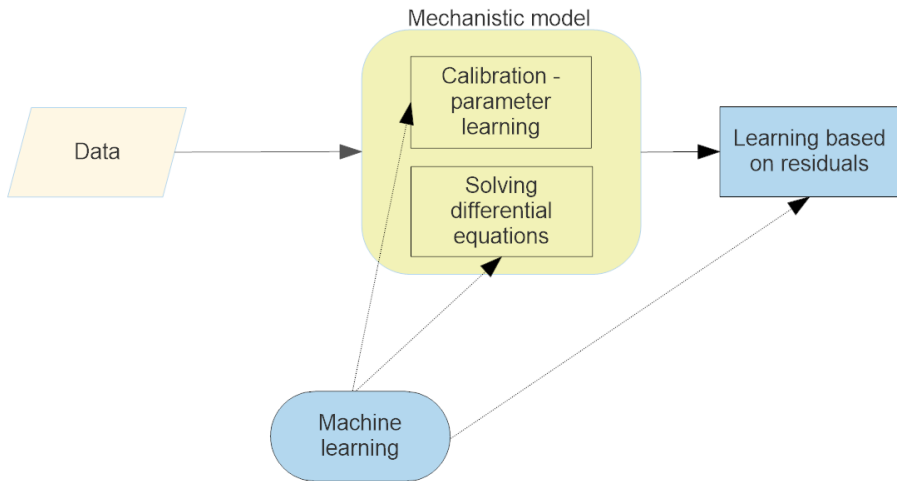


Figure: Machine learning flow [1]

Next, we will explore how one can integrate mechanistic models with machine learning.







## Training data

- Features
  - feature engineering
- Data augmentation
  - image transforms
  - simulations: generate a large amount of data from mechanistic models for training.

## Feature learning

- Unsupervised learning - knowledge can still be incorporated
- Variational autoencoders
  - VAEs jointly learn an inference model and a generative model, allowing them to infer latent variables from observed data.

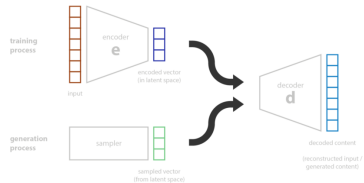
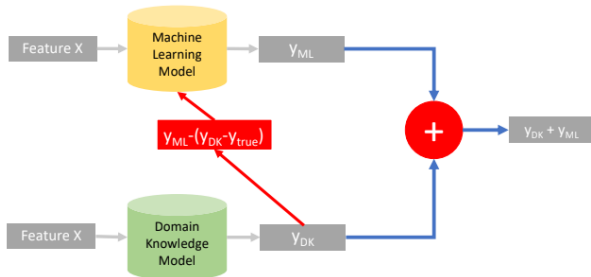


Figure: Understanding Variational Autoencoders (VAEs)

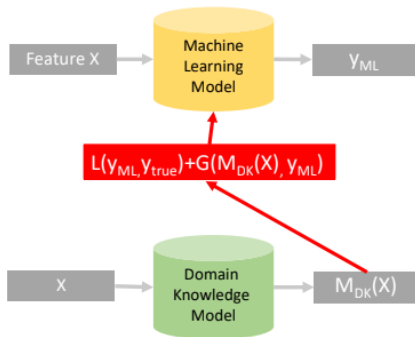
ML models the residuals of the domain knowledge model and tries to reduce the error between the mechanistic model output and the ground truth.



- $Y_{ML}$ : machine learning predicted label
- $Y_{DK}$ : domain knowledge predicted label
- $Y_{true}$ : ground truth label

Figure: Residual modeling [2]

- $Mass = Density \cdot Volume$ : ML does not know that this is not supposed to be violated
- Domain knowledge is into a loss function and performs regularization



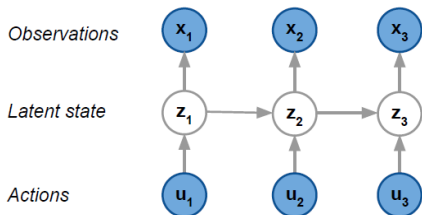
- function  $G()$  is regularization term: a measure of consistency between domain knowledge and predicted label
- function  $M_{DK}()$ : a domain knowledge transformation of feature X

Figure: Knowledge in the loss function [2]

- Model structure incorporates the mechanistic model
- We will introduce state-space models first and show how they can be integrated with RNNs
- Integration is done with variational autoencoders

State-space models:

- are numerically efficient to solve,
- can describe differential equations,
- allow for a more geometric understanding of dynamic systems, and
- form the basis for much of modern control theory



- Transition equation:

$$\mathbf{z}_t = \mathbf{A}_t \mathbf{z}_{t-1} + \mathbf{B}_t \mathbf{u}_t + \epsilon_t$$

- Emission equation:

$$\mathbf{x}_t = \mathbf{C}_t \mathbf{z}_t + \delta_t$$

Figure: Linear state-space model [3]

## Kalman filter:

- Kalman filter is optimal for linear Gaussian problems.
- Generalizes many common time-series models
- Strong modelling assumptions:
  - Linear transitions and emissions
  - Gaussian transitions and measurement noise

## Non-linear filters

- Extended Kalman filters (non-linear observation equation, Gaussian noise)
- Particle filters (non-linear, non Gaussian)
- Problems
  - Transition model still have difficulties handling complex non-linear dynamics
  - Does not capture long-term dependencies in data (Markov models)

## RNN

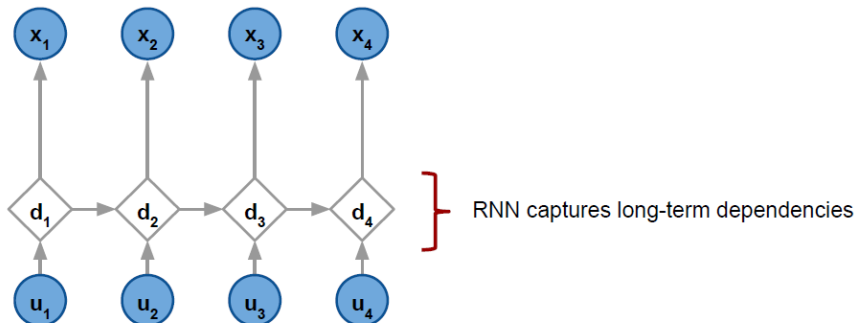


Figure: RNN [3]

## Non-linear SSM

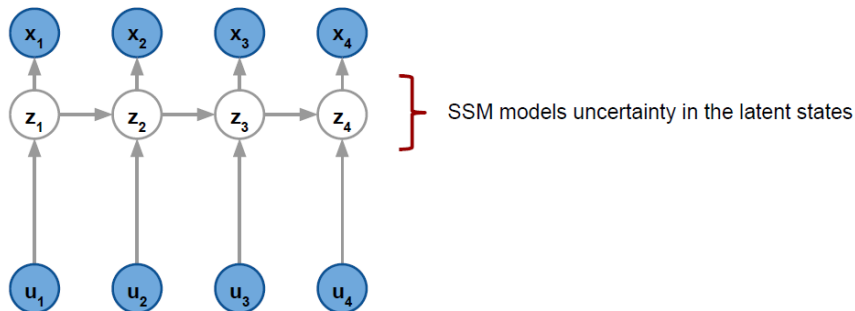


Figure: State-space model [3]



## SRNN

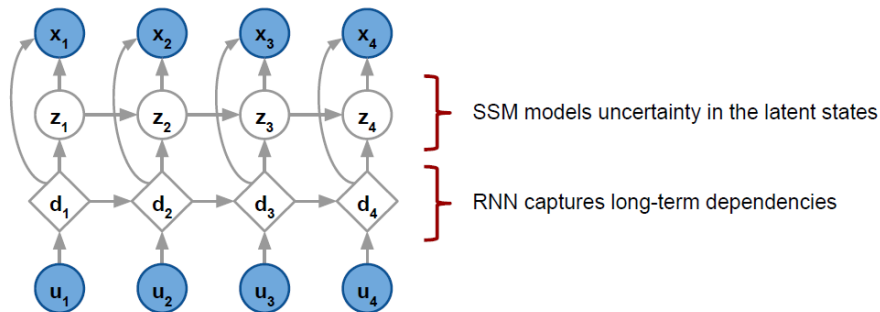


Figure: Merged RNN and state-space model [3]

Infer solutions to partial differential equations, and obtain physics-informed surrogate models [4]

- Neural networks can represent an arbitrary functions when given appropriate weights.
- Therefore it can approximate any arbitrary function that represents a solution of a differential equation:  $u = NN(x)$
- Assume that we are given a differential equation with boundary conditions.
- We can also find  $du/dx$ ,  $d^2u/dx^2$  through back-propagation.
- The goal is to minimize the mean square error loss formed by differential equation and boundary conditions using automated differentiation.

- Each data points is a random variable generated from multivariate normal distribution
- The relationship between random variables determines the shape of the latent function.
- Advantages:
  - Regression and prediction with confidence intervals [5]
  - Learning the parameters of the state space models or differential equations [6]
  - Time series where data is not uniformly sampled.
  - Allow for Bayesian optimization

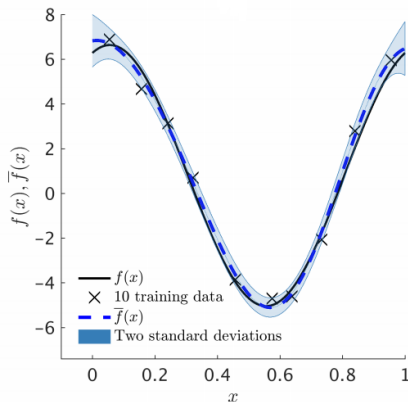


Figure: Gaussian process regression example [6]

- State-space models:  $dC_i/dt = \mu_i(t)C_i$ ,  
 $C_i$  and  $\mu_i$  are the concentrations and specific rates of the  $i^{th}$  species, respectively, and  $i$  represents viable cell density (Xv), concentration of glucose (GLC), lactate (LAC), glutamine (GLN), glutamate (GLU) and ammonia (NH4), and osmolality (Osm), and titer.
- $\mu_i$  is estimated based on ML
- $C_i$  is estimated using Extended Kalman filter

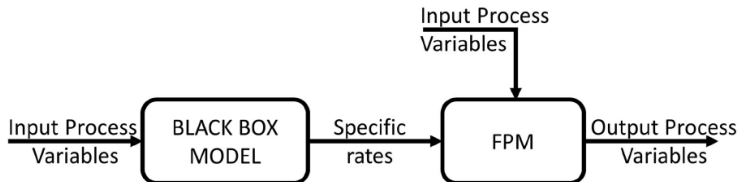


Figure: Hybrid state-space model (FPM is first-principle model) [7]

- Design a simulator that will allow us to simulate the bioprocess and bioreactor based on mechanistic or hybrid model [7]. This is important for:
  - digital twin
  - generating data for testing algorithms
- Merging mechanistic and ML models in this field has just started
  - there is great research opportunity to be first to apply some of these ML approaches on data from bioreactor.
- Gaussian processes

- Sequential decision making: Incorporating human knowledge into:
  - Reward
  - Policy and action selection
- Pre-training or intelligent initialization of the parameters of the ML model
  - Transfer learning

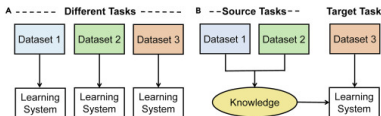


Figure: Transfer learning [8]

- Meta learning
  - learning from other processes
  - from our data collected using different sensors and in different ways

- [1] von Rueden L, Mayer S, Beckh K, Georgiev B, Giesselbach S, Heese R, et al. Informed Machine Learning – A Taxonomy and Survey of Integrating Knowledge into Learning Systems. 2020.
- [2] Jia X. ML applications in scientific domains; 2020.  
Available from:  
[https://drive.google.com/file/d/1WkIAX7vVQ6ehKvCQQCb13Lw\\_T7WG8aqY/view](https://drive.google.com/file/d/1WkIAX7vVQ6ehKvCQQCb13Lw_T7WG8aqY/view).
- [3] Fraccaro M, Sønderby SK, Paquet U, Winther O. Sequential Neural Models with Stochastic Layers. In: Proceedings of the 30th International Conference on Neural Information Processing Systems. NIPS'16. Red Hook, NY, USA: Curran Associates Inc.; 2016. p. 2207–2215.
- [4] Raissi M, Perdikaris P, Karniadakis GE. Physics Informed Deep Learning (Part I): Data-driven Solutions of Nonlinear Partial Differential Equations; 2017.
- [5] Swain SKLA P. Inferring time derivatives including cell growth rates using Gaussian processes. Nat Commun. 2016;7.

- [6] Raissi M, Perdikaris P, Karniadakis GE.  
Machine learning of linear differential equations using Gaussian processes.  
Journal of Computational Physics. 2017 Nov;348:683–693.  
Available from: <http://dx.doi.org/10.1016/j.jcp.2017.07.050>.
- [7] Narayanan H, Behle L, Luna MF, Sokolov M, Guillén-Gosálbez G, Morbidelli M, et al.  
Hybrid-EKF: Hybrid model coupled with extended Kalman filter for real-time monitoring and control of mammalian cell culture.  
Biotechnology and Bioengineering. 2020;117(9):2703–2714.  
Available from: <https://onlinelibrary.wiley.com/doi/abs/10.1002/bit.27437>.
- [8] Deng C, Ji X, Rainey C, Zhang J, Lu W.  
Integrating Machine Learning with Human Knowledge.  
iScience. 2020;23(11):101656.  
Available from: <http://www.sciencedirect.com/science/article/pii/S2589004220308488>.